sides $\alpha=0$, $\beta=0$, $\gamma=0$, $\delta=0$, and the *lengths* of the corresponding sides by a, b, c, and d. Let radius of circle be r. Then the equation of the line joining the middle points of the diagonals is $a\alpha-b\beta+c\gamma-d\delta=0$....(1) (Cf. Salmon's *Conics*, page 54, Ex. 5).

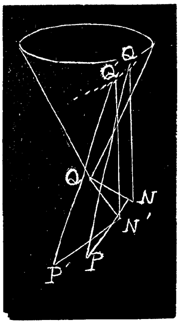
Putting $\alpha=\beta=\gamma=\delta=r$ we obtain $r(a-b+c-d)=0$, which is satisfied since $a+c=b+d$.

137. **Proposed by J. W. YOUNG, Oliver Graduate Scholar in Mathematics, Cornell University, Ithaca, N. Y.**

A right cone has its vertex in a horizontal plane, its axis being perpendicular to the plane. A string has one extremity attached to a point on the cone. The other extremity, P, of the string is kept in the plane, and the string is then wound around the cone, without being allowed to slip. Show that the spiral generated by P cuts all straight lines through the vertex at the same angle.

Solution by the PROPOSER.

Let P, P' be two points on the spiral; Q, Q' the corresponding points in the path of the string around the cone; N, N' the points where the perpendiculars from Q, Q' to the plane through the vertex O of the cone, cut the plane.



The right-angled triangles QNO, $Q'N'O$ have the angles QON and $Q'ON'$ equal; hence they are similar.

$$\therefore \frac{QN}{ON}=\frac{Q'N'}{ON'}\ldots.(1).$$

Again, since the string must not slip, it makes a constant angle with the plane.

$\therefore$ $\triangle\, QNP$ is similar to $\triangle\, Q'N'P'$.

$$\therefore \frac{PN}{QN}=\frac{P'N'}{Q'N'}\ldots.(2).$$

From (1) and (2),

$$\therefore \frac{PN}{ON}=\frac{P'N'}{ON'}\ldots.(3).$$

But the triangles ONP and $ON'P'$ are right-angled at N and N' (PN, $P'N'$ being the projections to tangents to the circular cone). From (3),

$\therefore$ $\triangle\, ONP$ is similar to $\triangle\, ON'P'$.

$\therefore$ $\measuredangle OPN=\measuredangle OP'N'$.

Observing that PN and $P'N'$ are normals to the spiral, the last equation states that the normals make a constant angle with rays through O. Q. E. D.

AVERAGE AND PROBABILITY.

90. **Proposed by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.**

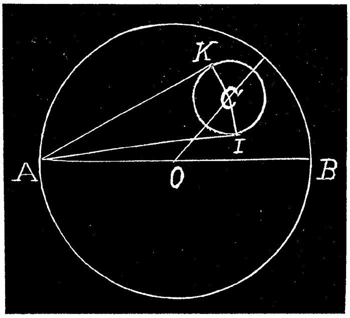
During a heavy rain storm a circular pond is formed in a circular field. If a man undertakes to cross the field in the dark, what is the chance that he will walk into the pond? [From *Byerly's Integral Calculus.*]

I. **Solution by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.**

Let O be the center of the circular field, and R its radius; C the center of

the circular pond and z its radius; $x=OC$, the distance from the center of the field to the center of the pond; $\angle BOC=\theta$, $\angle IAC=\phi$.

Then $AC=(R^2+x^2+2Rx\cos\theta)^{\frac{1}{2}}$ and $\phi=\sin^{-1}\left(\frac{z}{\surd(R^2+x^2+2Rx\cos\theta)}\right)$.



Then the probability that the man will walk into the pond for any particular value of ϕ is

$$\frac{2\phi.AK}{\pi AK}=\frac{2\phi}{\pi}.$$

The limits of z are 0 and $R-x$; of x, 0 and R; and of θ, 0 and π, and doubled.

Hence, the probability that the man walks into the pond is

$$p=\frac{2}{\pi}\frac{\int_0^\pi\int_0^R\int_0^{R-x}\sin^{-1}\left(\frac{z}{\surd(R^2+x^2+2Rx\cos\theta)}\right)d\theta.2\pi xdx.dz}{\int_0^\pi\int_0^R\int_0^{R-x}d\theta.2\pi xdx.dz}$$

$$=\frac{12}{\pi^2R^3}\int_0^\pi\int_0^R\int_0^{R-x}\sin^{-1}\left(\frac{z}{\surd(R^2+x^2+2Rx\cos\theta)}\right)d\theta.xdx.dz$$

$$=\frac{12}{\pi^2R^3}\int_0^\pi\int_0^R\left[z\sin^{-1}\left(\frac{z}{(R^2+x^2+2Rx\cos\theta)^{\frac{1}{2}}}\right)\right.$$

$$\left.+\sqrt{1-\frac{z^2}{(R^2+x^2+2Rx\cos\theta)}}\right]_0^{R-x}d\theta.xdx$$

$$=\frac{12}{\pi^2R^3}\int_0^\pi\int_0^R\left[(R-x)\sin^{-1}\left(\frac{R-x}{\surd(R^2+x^2+2Rx\cos\theta)}\right)+2\surd(Rx)\cos\tfrac{1}{2}\theta\right.$$

$$\left.-\surd(R^2+x^2+2Rx\cos\theta\right]d\theta.xdx$$

$$=\frac{12}{\pi^2R^3}\int_0^\pi\int_0^R(R-x)\tan^{-1}\left(\frac{R-x}{2\surd(Rx)\cos\frac{1}{2}\theta}\right)+\frac{12}{\pi^2R^3}\int_0^\pi\int_0^R[2\surd(Rx)\cos\tfrac{1}{2}\theta$$

$$-\surd(R^2+x^2+2Rx\cos\theta)]d\theta.xdx$$

$$=\frac{12}{\pi^2R^3}\int_0^\pi\left(\left[\tfrac{1}{6}x^2\tan^{-1}\left(\frac{R-x}{2\surd(Rx)\cos\frac{1}{2}\theta}\right)\right]_0^R\right.$$

$$\left.+\tfrac{1}{6}\int_0^R\frac{x^{\frac{3}{2}}R^{\frac{1}{2}}(R+x)(3R-2x)\cos\frac{1}{2}\theta}{R^2+x^2+2Rx\cos\theta}dx\right)d\theta$$

$$+\frac{12}{\pi^2 R^3}\int_0^\pi \left[\tfrac{4}{5}R^{\frac{1}{2}}x^{\frac{5}{2}}\cos\tfrac{1}{2}\theta-\tfrac{1}{3}(R^2+x^2+2Rx\cos\theta)^{\frac{3}{2}}+\tfrac{1}{2}R\cos\theta(x+2R\cos\theta)\right.$$

$$\left.\sqrt{(R^2+x^2+2Rx\cos\theta)}+\tfrac{1}{2}R^3\sin^2\theta\cos\theta\log[\sqrt{(R^2+x^2+2Rx\cos\theta)}+x+R\cos\theta]\right]_0^R d\theta$$

$$=\frac{2}{\pi^2R^3}\int_0^R\int_0^\pi \frac{x^{\frac{3}{2}}R^{\frac{1}{2}}(R+x)(3R-2x)\cos\frac{1}{2}\theta}{R^2+x^2+2Rx\cos\theta}dxd\theta+\frac{12}{\pi^2R^3}\int_0^\pi\frac{R^3}{30}\Big[24\cos\tfrac{1}{2}\theta$$

$$-140\cos^3\tfrac{1}{2}\theta+120\cos^5\tfrac{1}{2}\theta+10-15\cos^2\theta+15\sin^2\theta\cos\theta\log\left(\frac{1+\cos\frac{1}{2}\theta}{\cos\frac{1}{2}\theta}\right)\Big]d\theta$$

$$=\frac{2}{\pi^2R^3}\int_0^R\left[\frac{1}{\sqrt{-1}}x(3R-2x)\tan^{-1}\left(\frac{2\sqrt{(-Rx)}}{R+x}\sin\tfrac{1}{2}\theta\right)\right]_0^\pi dx+\frac{2}{5\pi^2}\int_0^\pi\Big[24\cos\tfrac{1}{2}\theta$$

$$-140\cos^3\tfrac{1}{2}\theta+120\cos^5\tfrac{1}{2}\theta+10-15\cos^2\theta+15\sin^2\theta\cos\theta\log\left(\frac{1+\cos\frac{1}{2}\theta}{\cos\frac{1}{2}\theta}\right)\Big]d\theta$$

$$=\frac{2}{\pi^2R^3}\int_0^R\frac{1}{\sqrt{-1}}\left[x(3R-2x)\tan^{-1}\left(\frac{2\sqrt{(-Rx)}}{R+x}\right)\right]dx-\frac{32}{\pi^2}$$

$$=\frac{2}{\pi^2R^3}\left(\left[\frac{x^2}{6\sqrt{-1}}(9R-4x)\tan^{-1}\left(\frac{2\sqrt{(-Rx)}}{R+x}\right)\right]_0^R\right.$$

$$\left.-\frac{\sqrt{(-R)}}{6\sqrt{(-1)}}\int_0^R\frac{x^{\frac{3}{2}}(9R-4x)}{R-x}dx\right)-\frac{32}{\pi^2}=\frac{5}{3\pi^2\sqrt{-1}}\tan^{-1}(\sqrt{-1})$$

$$-\frac{1}{3\pi^2R^{\frac{5}{2}}}\int_0^R(4x^{\frac{3}{2}}-5Rx^{\frac{1}{2}}-5R^2x^{-\frac{1}{2}}+\frac{5R^2}{\sqrt{x}\,(R-x)})dx-\frac{32}{\pi^2}$$

$$=\frac{5}{3\pi^2\sqrt{-1}}\tan^{-1}(\sqrt{-1})-\frac{1}{3\pi^2R^{\frac{5}{2}}}\Big[\tfrac{8}{5}x^{\frac{5}{2}}-\tfrac{10}{3}Rx^{\frac{3}{2}}-10R^2x^{\frac{1}{2}}$$

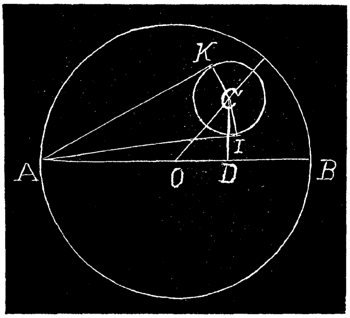
$$+\frac{5R^{\frac{3}{2}}}{\sqrt{-1}}\tan^{-1}\sqrt{\left(-\frac{x}{R}\right)}\Big]_0^R-\frac{32}{\pi^2}=\frac{176}{45\pi^2}-\frac{32}{15\pi^2}=\frac{16}{9\pi^2}.$$

J. M. Colaw, M. E. Graber, and Walter H. Drane get the result $p=l/L$, where l is the circumference of the pond, and L the circumference of the field. But the circumference of the pond varies from 0 to $2\pi R$ where R is the radius of the field. Assuming as they do, that the chance of the man walking into the pond is the totality of all lines crossing both the field and pond divided by the totality of all lines crossing the field, the result l/L is not satisfactory, since l is variable, as already stated.

The problem, of course, is indefinite, since the law of variation of the several events are not definitely stated, and thus as many different results may be obtained as there are possible interpretations of the meaning of the problem.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let O be the center of the field, radius r; C the center of the pond, radius c. The chance p, of walking into the pond is $\angle KAI/\pi=\varphi/\pi$.



Let $(x-a)^2+(y-b)^2=c^2$ be the equation to the pond ; $y=m(x+r)$, the equation to a line meeting the pond.

When $y=m(x+r)$ is tangent to the pond we have

$$\tan BAE=m_2=-\frac{b(a+r)+c\sqrt{b^2-c^2+(a+r)^2}}{c^2-(a+r)^2}$$

$$\tan BAF=m_1=-\frac{b(a+r)-c\sqrt{b^2-c^2+(a+r)^2}}{c^2-(a+r)^2}$$

$$\therefore\ \tan EAF=\frac{m_1-m_2}{1+m_1m_2}=\frac{2c\sqrt{[b^2-c^2+(a+r)^2]}}{[b^2-c^2+(a+r)^2]-c^2}=\tan\varphi.$$

$$\therefore\ \varphi=\tan^{-1}\left(\frac{2c\sqrt{[b^2-c^2+(a+r)^2]}}{[b^2-c^2+(a+r)^2]-c^2}\right)=2\tan^{-1}\left(\frac{c}{\sqrt{[b^2-c^2+(a+r)^2]}}\right).$$

If $a=\rho\cos\theta$, $b=\rho\sin\theta$, $\varphi=2\tan^{-1}\left(\frac{c}{\sqrt{(\rho^2+r^2+2\rho r\cos\theta)}}\right)$.

$$\therefore\ p=\frac{\frac{1}{\pi}\int_0^\pi\int_0^r\int_0^{r-\rho}\varphi\rho\,d\theta\,d\rho\,dc}{\int_0^\pi\int_0^r\int_0^{r-\rho}\rho\,d\theta\,d\rho\,dc}=\frac{6}{\pi^2r^3}\int_0^\pi\int_0^r\int_0^{r-\rho}\varphi\rho\,d\theta\,d\rho\,dc$$

$$=\frac{12}{\pi^2r^3}\int_0^\pi\int_0^r\left[(r-\rho)\tan^{-1}\left(\frac{r-\rho}{2\sqrt{(r\rho)}\cos\frac{1}{2}\theta}\right)+2\sqrt{(r\rho)}\cos\tfrac{1}{2}\theta\right.$$
$$\left.-\sqrt{(\rho^2+r^2+2r\rho\cos\theta)}\right]\rho\,d\theta\,d\rho.$$

$$\frac{12}{\pi^2r^3}\int_0^\pi\int_0^r[2\sqrt{(r\rho)}\cos\tfrac{1}{2}\theta-\sqrt{(\rho^2+r^2+2r\rho\cos\theta)}]\rho\,d\theta\,d\rho$$

$$=\frac{2}{5\pi^2}\int_0^\pi\left[24\cos\tfrac{1}{2}\theta-140\cos^3\tfrac{1}{2}\theta+120\cos^5\tfrac{1}{2}\theta+10-15\cos^2\theta\right.$$
$$\left.+15\sin^2\theta\cos\theta\log\left(\frac{1+\cos\frac{1}{2}\theta}{\cos\frac{1}{2}\theta}\right)\right]d\theta=-\frac{32}{15\pi^2}.$$

$$\frac{12}{\pi^2r^3}\int_0^\pi\int_0^r(r-\rho)\tan^{-1}\left(\frac{r-\rho}{2\sqrt{(r\rho)}\cos\frac{1}{2}\theta}\right)\rho\,d\theta\,d\rho$$

$$=\frac{2}{\pi^2r^3}\int_0^\pi\int_0^r\frac{\rho\sqrt{(r\rho)}(r+\rho)(3r-2\rho)\cos\frac{1}{2}\theta d\rho d\theta}{\rho^2+r^2+2r\rho\cos\theta}$$

$$=\frac{2}{\pi^2r^3\sqrt{-1}}\int_0^r\rho(3r-2\rho)\tan^{-1}\left(\frac{2\sqrt{(-r\rho)}}{r+\rho}\right)d\rho\,.$$

$$\frac{2}{\pi^2r^3\sqrt{-1}}\int_0^r\rho(3r-2\rho)\tan^{-1}\left(\frac{2\sqrt{(-r\rho)}}{r+\rho}\right)d\rho=\frac{5}{3\pi^2\sqrt{-1}}\tan^{-1}\sqrt{-1}$$

$$-\frac{1}{3\pi^2r^3}\int_0^r\frac{\sqrt{(r\rho)}(9r\rho-4\rho^2)d\rho}{r-\rho}=\frac{5}{3\pi^2\sqrt{-1}}\tan^{-1}\sqrt{-1}$$

$$-\frac{1}{3\pi^2r^3}\left[\tfrac{8}{5}\rho^2\sqrt{(r\rho)}-\tfrac{10}{3}r\rho\sqrt{(r\rho)}-10r^2\sqrt{(r\rho)}+\frac{5r^3}{\sqrt{-1}}\tan^{-1}\sqrt{-\frac{\rho}{r}}\right]_0^r$$

$$=\frac{5}{3\pi^2\sqrt{-1}}\tan^{-1}\sqrt{(-1)}+\frac{176}{45\pi^2}-\frac{5}{3\pi^2\sqrt{-1}}\tan^{-1}\sqrt{(-1)}=\frac{176}{45\pi^2}\,.$$

$$\therefore\ p=\frac{176}{45\pi^2}-\frac{32}{15\pi^2}=\frac{16}{9\pi^2}.$$

EDITORIALS.

The Summer Meeting, 1901, of the American Mathematical Society will be held at Cornell University.

Several Complete sets of the MONTHLY are wanted. Any reader possessing a complete set in good condition and desiring to sell it, should write to the editor stating the price desired.

The MONTHLY is mailed on the 28th of each month, and should reach the greater part of its readers within five days after it is mailed. Subscribers should notify us promptly of any change of address, or of any failure to receive their copies.

Dr. G. A. Miller has just been awarded the prize of $260. from the Cracow Academy of Austria for his work in the Theory of Groups. We rejoice with Dr. Miller in this well merited recognition of his work. Dr. Miller is a young mathematician, yet his contributions to *The American Journal of Mathematics*, *The Annals of Mathematics*, *The Messenger of Mathematics*, *The Proceedings of the London Mathematical Society*, *The Transactions of the American Mathematical Society*, the MONTHLY, and many other mathematical journals in both hemispheres, are very numerous.